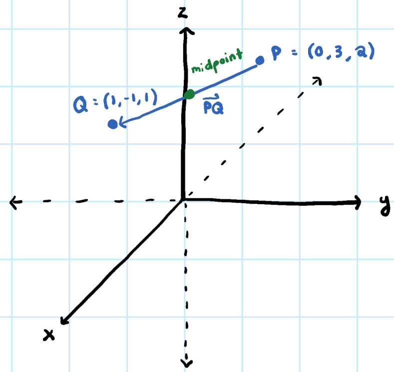
Review Midterm 2

Wednesday, May 24, 2023 8:50 AM



points & vectors:

- given P, Q, find $\vec{PQ}$

$$\vec{PQ} = \langle 1-0, -1-3, 1-2\rangle = \langle 1, -4, -1\rangle$$

- midpoint: sum components & $\div 2$ or $\vec{OP} + \frac{1}{2}\vec{PQ}$

$$\text{midpoint} = \left(\frac{0+1}{2}, \frac{3+-1}{2}, \frac{2+1}{2}\right) = \left(\frac{1}{2}, 1, \frac{3}{2}\right)$$

planes & spheres:

1) equation of plane: $\{ax + by + cz = d\}$ — $\langle a,b,c\rangle$ = perpendicular to plane! a, b, c, d are #

e.g. $\pi := \{x - 2y + 3z = 16\} \longrightarrow$ perpendicular $= \langle 1, -2, 3\rangle$ — doesn't depend on d

* if variable (x, y, or z) not there → coefficient = 0 *

2) equation for sphere of center $= (-7, 0, 3)$ & radius $R = 9$

$$(x - -7)^2 + (y-0)^2 + (z-3)^2 = 9^2$$
$$(x+7)^2 + y^2 + (z-3)^2 = 81$$

* master intersection of plane & plane and plane & sphere * *

plane intersecting sphere = circle

plane intersecting ball = disk

plane intersecting plane = line or parallel

planes (again ☺):

$\{ax + by + cz = d\}$ ← $\vec{n} = \langle a,b,c\rangle$, $d := \vec{OP}\cdot\vec{n} = ax + by + cz$

3 points P, Q, R in space → 2 vectors $\vec{u}$ & $\vec{v}$ in plane + point → perpendicular vector $\vec{n}$ + point in plane P

$\vec{u} = \vec{PQ}$ & $\vec{v} = \vec{PR}$

$\vec{n} = \vec{u}\times\vec{v}$

vector operations: $\vec{u}, \vec{v}$ = vectors

$\vec{u} = \langle 3, 1, 0\rangle$ $\vec{v} = \langle 0, -5, 7\rangle$

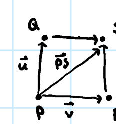
- dot product: (#) $\vec{u}\cdot\vec{v} = u_1v_1 + u_2v_2 + u_3v_3 = (3\cdot 0) + (1\cdot -5) + (0\cdot 7) = -5 \longrightarrow$ size of $\mathrm{proj}_{\vec{v}}\vec{u}$
  - $\vec{u} \perp \vec{v} \longleftrightarrow \vec{u}\cdot\vec{v} = 0$
- cross product: (vector) $\vec{u}\times\vec{v} = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \begin{vmatrix} i & j & k \\ 3 & 1 & 0 \\ 0 & -5 & 7 \end{vmatrix} = \langle 7-0, -(21-0), -15-0\rangle = \langle 7, -21, -15\rangle$
  - $\vec{u} \parallel \vec{v} \longleftrightarrow \vec{u}\times\vec{v} = \langle 0,0,0\rangle = \vec{0}$

angle formulas: $\vec{u}, \vec{v}$ vectors & θ angle

- $\vec{u}\cdot\vec{v} = |\vec{u}|\cdot|\vec{v}|\cos\theta \longrightarrow$ compute θ (or $\cos\theta$)
- $|\vec{u}\times\vec{v}| = |\vec{u}|\cdot|\vec{v}|\sin\theta \longrightarrow$ area of parallelogram of $\vec{u}$ & $\vec{v}$ $= |\vec{u}\times\vec{v}|$

need to be length bc $\vec{u}\times\vec{v}$ = vector & $|\vec{u}|\cdot|\vec{v}|\sin\theta$ = #

ex) 4 points $P = (0,1,0)$, $Q = (0,1,1)$, $R = (2,1,0)$, $S = (2,1,1)$



solution:

- choose $\vec{u} = \vec{PQ} = \langle 0,0,1\rangle$ $\vec{PS} = \langle 2, 0, 1\rangle$

$\vec{v} = \vec{PR} = \langle 2, 0, 0\rangle$

- $\vec{u}\times\vec{v} = \begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ 2 & 0 & 0 \end{vmatrix} = \langle 0, 2, 0\rangle$ = area is length

$= \sqrt{0^2+2^2+0^2} = \boxed{2}$

* review distances *

trajectories:

$\vec{r}(t) = \langle \cos(4t), \sin(7t), t^3 \rangle$

$\vec{r}(0) = \langle 1, 0, 0 \rangle$ → initial position @ $t=0$

length of $\vec{v}$ = speed

$\vec{v}(t) = \vec{r}(t)' = \langle -4\sin(4t), 7\cos(7t), 3t^2 \rangle$

$\vec{a}(t) = \vec{r}(t)'' = \langle -16\cos(4t), -49\sin(7t), 6t \rangle$

* review integrals *